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FHWA/IN/JHRP-94/8
Final Report

IMPROVED STRATEGIES FOR DEPLOYING VEHICLE-ACTUATEDCONTROL AT ISOLATED SIGNALIZED INTERSECTIONS

Michael J. Cassidy
Yu-hao Chuang
Jeff Vitale

PURDUE UNIVERSITY

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16. Abstract

This research has sought to demonstrate potential benefits from deploying enhanced vehicle actuation strategies at isolated signalized intersections. The work has exploited microscopic, stochastic simulation to evalnate impacts of enhanced vehicle-actuated (VA) control schemes for an array of operating conditions. Simulated outcomes (i.e. average vehicle delays) generated under the enhanced strategies were compared with outcomes resulting from more "conventional" VA control policies.

Findings from this work suggest that substantial delay reduction generally occurs by exploiting VA strategies which seek to]) facilitate the use of the clearance interval by discharging vehicles, 2) shorten the duration of the required clearance interval by only serving, to the extent possible, queued vehicles and 3) evaluate gaps in individual traffic streams. The enhanced VA strategies described and tested in this research are inconsistent with conventional practice. Nonetheless, these enhanced schemes do not compromise traffic safety as motorists legally entitled to enter the intersection are always allocated clearance interval of sufficient duration.

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IMPLEMENTATION REPORT

This research has sought to demonstrate potential benefits from deploying enhanced vehicle actuation strategies at isolated signalized intersections. The work has exploited microscopic, stochastic simulation to evaluate impacts of enhanced vehicle-actuated (VA) control schemes for an array of operating conditions. Simulated outcomes (i.e. average vehicle delays) generated under the enhanced strategies were compared with outcomes resulting from more "conventional" VA control policies.

To calibrate the microscopic, stochastic simulation model, field data of vehicle dissipation headways were simulated by stopping probability table which was recalibrated from field data. Speed variations between vehicles are not so apparent that deterministic value is used for simplicity. To determine the simulation time, the relaxation time of the system reached the steady state is identified through statistical method. Also the sample size (the number of simulation run) is determined by statistical method under specified level of confidence.

Some desirable attributes of VA control are introduced as the theoretical base of enhanced VA strategies. The desirable attributes results in the chain reaction which can promote a substantial reduction in vehicle delay.

Simulation modes from idealized two one-way streets to realistic four two-way streets are simulated. The simulated outcomes (average delay) are compared between "conventional" VA strategies and "enhanced" VA strategies under various traffic conditions.

Findings from this work suggest that substantial delay reduction generally occurs by exploiting VA strategies which seek to 1) facilitate the use of the clearance interval by discharging vehicles, 2) shorten

the duration of the required (i.e., safe) clearance interval by only serving, to the extent possible, queued vehicles and 3) evaluate gaps in individual traffic streams.

The enhanced VA strategies described and tested in this research are inconsistent with conventional practice. Nonetheless, these enhanced schemes do not compromise traffic safety as motorists legally entitled to enter the intersection are always allocated clearance intervals of sufficient duration.

Since changes in policy and hardware would be required, field implementation trials should be conducted. The feasibility of adding the following feature should be considered. A wide area detector can be focused on the conflict areas of the center of the intersection. This detector would extend the all-red indication as long as vehicles remained in the intersection (up to some maximum value, 7 seconds, for example). This would provide additional assurance that right angle collisions would not occur while allowing the more efficient shortened yellow intervals.

Since Greenfield and LaPorte Districts operate the majority of INDOT's traffic signals, they should be encouraged to pursue these changes. All of the traffic signal equipment manufacturers should also be sent this report and asked for their input and cooperation.

ACKNOWLEDGMENTS

The researchers wish to thank the JHRP advisory committee Messrs. Daniel Shamo, INDOT, Carl Tuttle, INDOT and Ed Ratulowski, FHWA for their support throughout this project.

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1. INTRODUCTION

A vehicle-actuated (VA) signalized intersection is one in which detectors are located in some or all travel lanes. These detectors sense the presence of vehicles. Information concerning vehicle presence is, in turn, used by the signal controller to determine appropriate phase times and cycle lengths. In theory, vehicle actuation facilitates more efficient operation than fixed-time control in that the appropriate durations for signal indications can be allocated to serve time-variant demands each cycle.

Realizing these operational benefits, however, requires the exploitation of efficient VA strategies. Efficient control schemes are documented in the literature [Newell, 1988]. Yet it appears that conventional engineering practice consistently utilizes VA control in sub-optimal means.

1.1. Research Objective

The authors propose that the practice of deploying "less-than-efficient" VA control may, to some extent, reflect misunderstanding (or perhaps skepticism) concerning the benefits achievable by using more efficient strategies. The objective of this thesis, then, is to demonstrate the

potential benefits (i.e. vehicle delay reduction) from deploying enhanced VA strategies.

1.2. Research Scope

In this work, the operational performance at isolated signalized intersections controlled using a number of enhanced VA strategies are compared with intersection performance under more "conventional" VA methods. Comparisons are made for an array of specified operating conditions (e.g. demand rates, approach speeds, geometrics).

To facilitate the evaluation of numerous operating conditions and control strategies, our work has exploited computer simulation. A microscopic stochastic simulation model has been developed and calibrated to emulate signalized intersection operating conditions.

To concisely illustrate the potential benefits of enhanced VA strategies, the study has adopted a range of operating conditions to be evaluated. Although VA control can significantly enhance operation at "low flow" intersections, the costs of vehicle actuation may not warrant deployment at such locations. At "high demand" (i.e. over-saturated) intersections, VA control typically displays maximum green times, and as such, provides little advantage over fixed-time control. Thus, this research has assessed operation under moderately high demand rates ranging from

480 to 800 vehicles per hour (vph) per lane. Average free-flow approach speeds range from 30 to 50 mph.

The work seeks to illustrate (in somewhat general terms) potential impacts of specific VA strategies. As such, evaluation has been limited to through-moving (i.e. non-turning) traffic streams. This limitation has greatly simplified the development of the simulation model yet serves to demonstrate relevant issues.

1.3. Research Contribution

The authors do NOT assume credit for developing the enhanced VA strategies described herein. To the contrary, the enhanced control methods evaluated in this work have been previously documented by G.F. Newell in his monograph on the *Theory of Highway Traffic Signals* [Newell, 1988]. Thus, the contribution of this study is not to introduce improved control methods, but rather to demonstrate and, to some extent, quantify their potential impacts.

1.4. Report Scope

Section 1 has sought to define the research objectives.

The second section of this report describes the computer simulation model developed and used for evaluating VA control strategies. Included here is discussion on the collection and analysis of empirical data for model

validation and the methods used to carry-out simulation experiments.

Section 3 evaluates VA control alternatives (by presenting simulated outcomes). To a large extent, appropriate VA strategies vary with operating conditions (e.g. approach geometrics, free-flow speeds etc.) and there are a number of issues involved in selecting the preferred strategy for any given scenario. The format adopted for Section 3 represents an effort to concisely convey these issues. The third section begins with general background regarding desirable "attributes" of VA control. This section then describes application of VA strategies for a number of operating conditions. For each scenario 1) considerations associated with efficient VA control are described, 2) simulations are performed for both enhanced and conventional VA strategies and 3) simulated outcomes are presented.

Conclusions are presented in the fourth and final section.

2. SIMULATION MODEL

A microscopic, stochastic computer simulation model was developed using the SLAM II simulation language [Pritsker, 1986] and subsequently used for assessing intersection performance.

2.1. Model Features

As previously noted, this work has evaluated impacts of VA strategies on through-moving traffic streams. The simulation model, therefore, does not replicate the behavior of turning traffic.

To capture the stochastic characteristics of vehicle passage times over detectors, vehicle arrival and queue discharge headways are generated subject to their observed distributions. With this stochastic component, the model is able to treat vehicle acceleration as instantaneous without compromising reliability. We note that delay, the primary performance measure, is independent of the precise characteristics of vehicles' trajectories [Newell, unpublished].

An additional stochastic component of intersection operation is the manner in which discharging vehicles

respond to clearance intervals. The simulation model exploits a Probit function [Ben-Akiva & Lerman, 1985] to generate the probability that a vehicle (i.e., motorist) stops in response to a yellow interval as a function of its present speed and distance from the stop bar.

Of less significance to VA operation is the variation in vehicle speeds arriving to, and departing from, the intersection approach. These speeds would generally exhibit a relatively small range. Thus, the model avoids the complexity of emulating the interactions of fast- and slow-moving vehicles by exploiting deterministic (specified) values of arrival and queue discharge speeds.

The simulation model consists of two primary components:

1. The replication of vehicular movement (including motorist response to signal control), and
2. The representation of signal operation given specified strategies and stochastically-generated vehicle actuations.

Figure 2.1. schematically diagrams the link and node structure used to emulate vehicle movements. A node is used to represent the time-variant location of a vehicle(and/or the fixed location of a detector). Links represent the path from one node to the next. A link length is coded as 24 feet, the presumed average space occupied by a queue vehicle. Thus, in Figure 2.1., vehicles enter the "system"

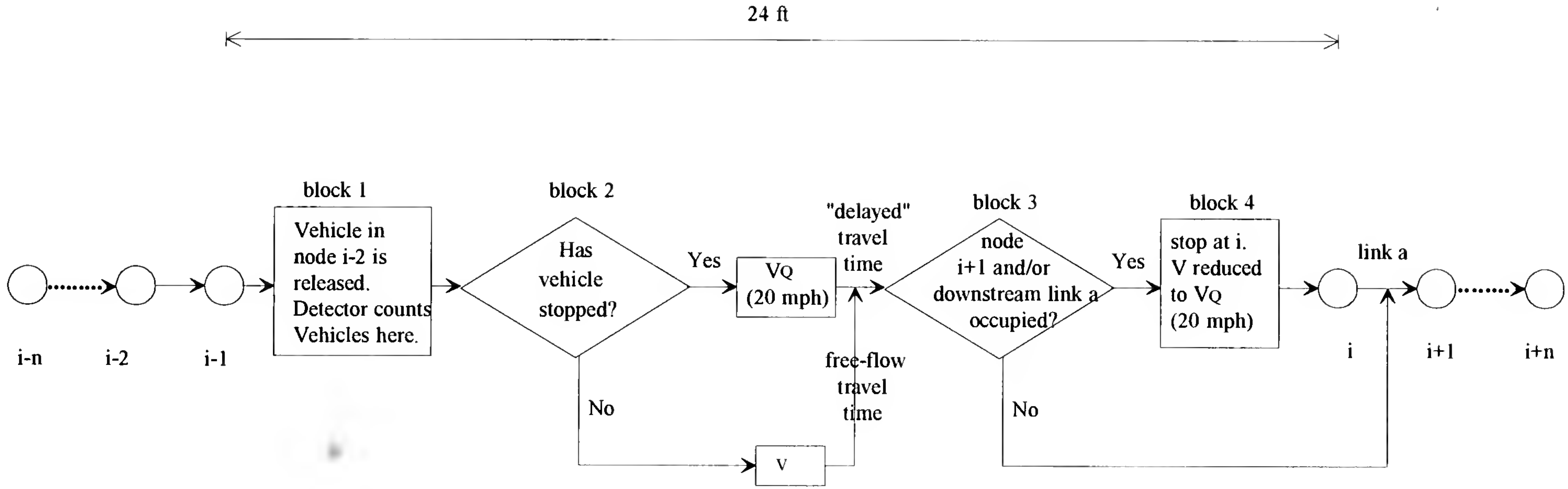

Figure 2.1. The Basic Structure of Traffic Flow Network

at node i-n and exit (i.e. enter the intersection) at node i+n.

In Figure 2.1., the link connecting nodes i-1 and i is presented in detail to illustrate program logic. We assume node i-1 marks the location of a detector. When a vehicle arrives to "block 1", thereby creating a vacancy in node i-1, any upstream vehicle presently occupying node i-2 moves forward. Further upstream vehicles likewise advance in this manner. The detector, presumed located in block 1, "counts" the vehicle as it passes and records the arrival time.

The vehicle is next passed to "block 2" where the program evaluates the status of the downstream link and node. Where downstream link and node are vacant, the vehicle advances forward. When a vehicle is required to stop(i.e., the downstream link and node are occupied), it eventually moves forward (following a downstream vacancy) subject to empirically observed distributions of queue "start-up" headways and specified queue discharge speed. These dynamics provide a reasonable replication of vehicle trajectories.

The traffic signal's operation, illustrated in Figure 2.2., is modeled as a downstream-most node. Where the vehicle queue has over-run the detector(s) at the onset of green, the logic automatically assigns a minimum green time and subsequently searches for a "critical gap" (β) in the traffic stream. Where the green is displayed prior to the queue over-running the detector, sufficient green time is

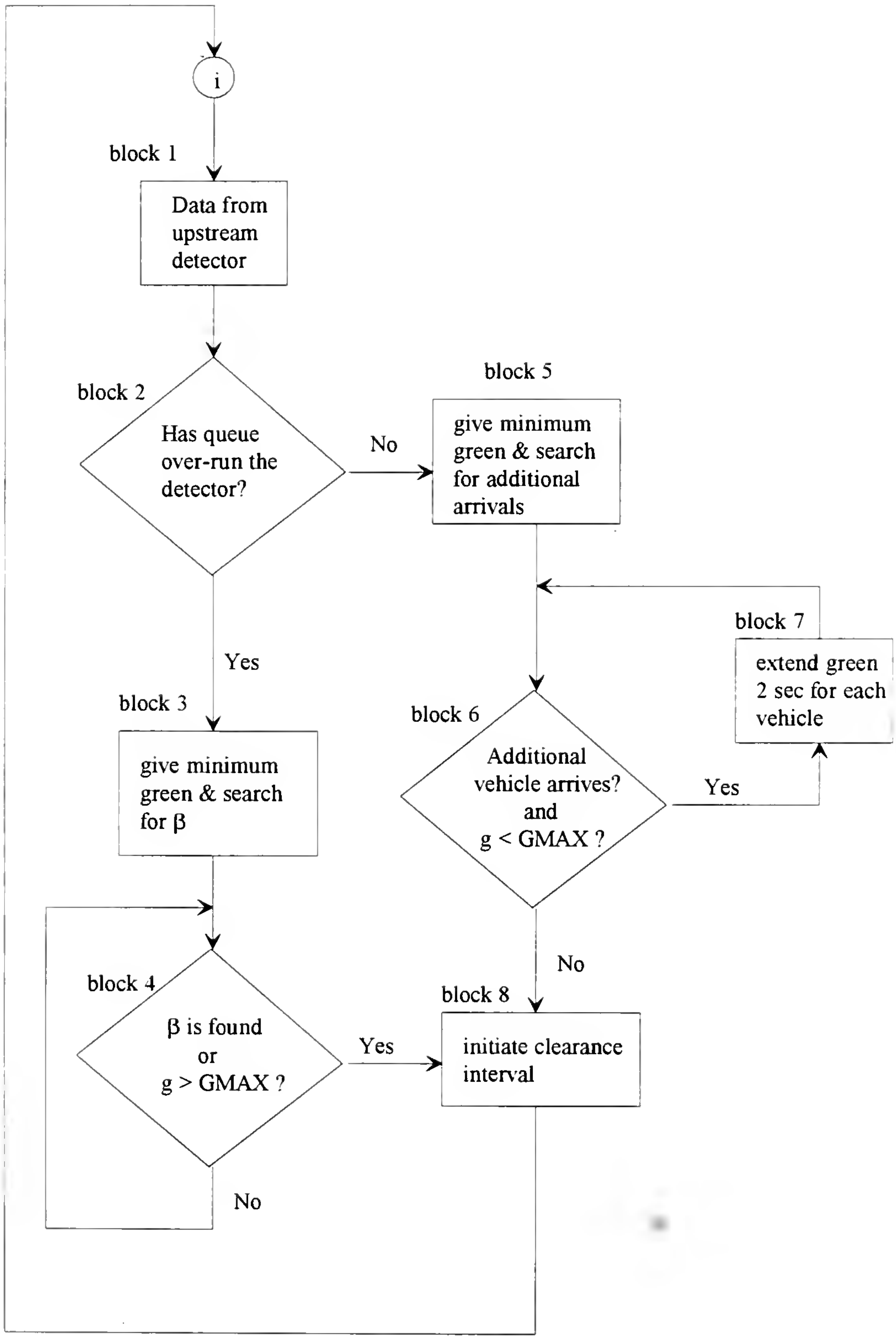

Figure 2.2. The Basic Structure of Signal Operation Network

allocated to serve the existing queue with additional green extensions to serve subsequent arrivals. The details of all signal allocation strategies are presented in Section 3.

2.1.1. Modeling Motorist Response to Clearance Interval

The simulation model emulates driver reaction to the yellow interval by capturing the variable behavior of drivers. To emulate this behavior, we re-calibrated a Probit function originally estimated using a data base reflecting 1,000 observations at 9 high-speed signalized intersections in the State of Kentucky [Sheffi & Mahmassani, 1981].

The original observations reflect the binary decision (i.e., the choice to stop or proceed through the intersection at the onset of yellow) of motorists in the presence of no downstream queuing. As such, the function, as originally estimated, does not represent the "snappier" operation simulated in our experiments. We therefore re-calibrated the function using the following logic:

The minimum and legal stopping distances were defined according to presumed driver deceleration rates of 10 and 8 ft/sec^2, respectively. We further adopted Zegeer's [1977] definitions of minimum and legal stopping distances as those locations were 10 and 90 percent of drivers stop, respectively, in response to yellow initiation.

With these guidelines, we re-estimated the coefficients of the Probit function derived by Sheffi & Mahmassani

[1981]. As displayed in Table 2.1., we thus established the probabilities that a vehicle stops in response to yellow initiation as a function of its speed and location on the approach (expressed as queue position or "node number").

Table 2.1. Probability of Stopping at Yellow Initiation

	Queue Position															
Speed(mph)	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16
20	0	0.16	0.99	1.0	1.0	1.0	1.0	1.0	1.0	1.0	1.0	1.0	1.0	1.0	1.0	1.0
30	0	0	0.12	0.86	0.98	1.0	1.0	1.0	1.0	1.0	1.0	1.0	1.0	1.0	1.0	1.0
35	0	0	0	0	0	0.20	0.88	1.0	1.0	1.0	1.0	1.0	1.0	1.0	1.0	1.0
40	0	0	0	0	0	0	0.12	0.32	0.90	1.0	1.0	1.0	1.0	1.0	1.0	1.0
50	0	0	0	0	0	0	0	0	0	0	0.01	0.02	0.26	0.79	0.99	1.0

The simulation model uses the estimated probabilities in Table 2.1. and the inverse method to generate the "stop/go" decision. Random values are generated from the [0,1] uniform distribution. Whenever the probability of stopping (Table 2.1.) exceeds the randomly generated number, the (simulated) vehicle stops in response to the yellow interval. We note that this formulation allows the possibility of "law breakers" who enter the intersection although "legally" required to stop.

2.1.2. Measures of Performance

The primary measure of performance generated by the simulation model is average vehicle delay. Delay is computed as the difference between the actual travel time on the intersection approach and the "desired" travel time given a specified free-flow speed. Average delay is merely computed as the sum of individual delays divided by the total arrival number. Average delays are computed for each intersection approach and for the overall intersection.

Additional measures generated by the "event-based" simulation model include:

1. Average and total travel times;
2. The percentage of vehicles required to stop;
3. The percentage of cycles where vehicle queues over-run the detectors by the green initiation time.

2.2. Empirical Calibration of Simulation Model

The average vehicle free-flow approach speed represents a user-specified value. An adequate value for the average discharge speed of a "fully accelerated" queue was field-measured (in a floating car) to be 20 mph. Finally, the arrival of vehicles on an isolated intersection approach is known to conform to (or at least closely approximate) a Poisson distribution [May, 1990]. As such, the only random variables requiring empirical calibration were the queue discharge headways.

Empirical observations were measured in two "through" lanes at a signalized intersection in Lafayette, Indiana. The site is illustrated in Figure 2.3.. Queue discharge headways were recorded over numerous cycles using a lap-top computer.

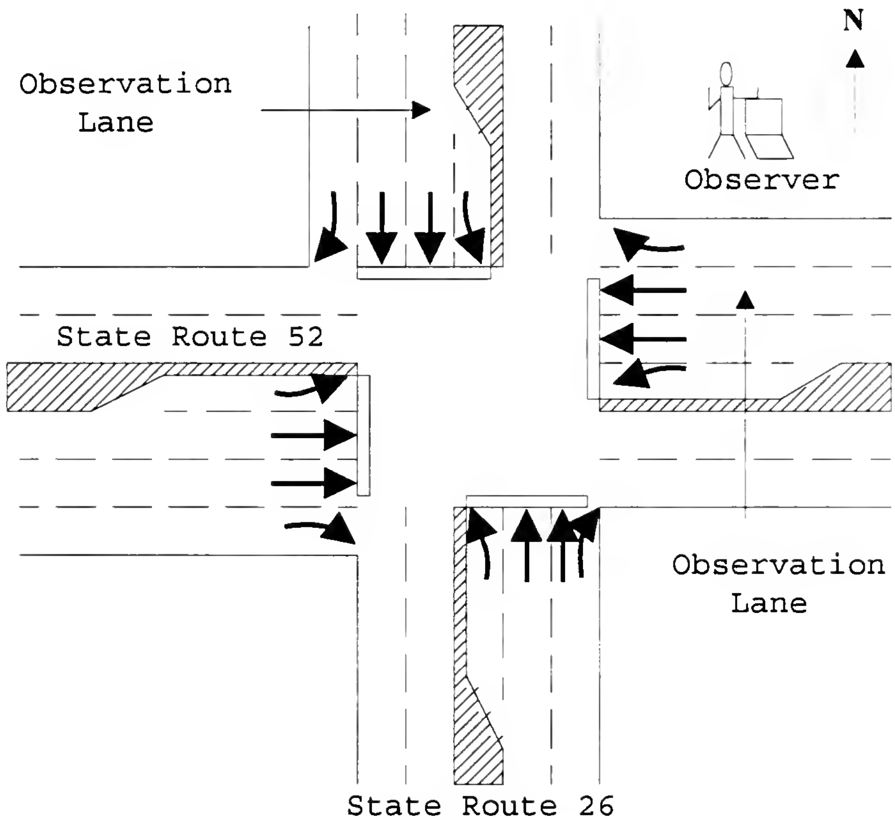

Figure 2.3. Data Collection Site

These discharge headways are known to vary as a function of position within the queue [TRB, 1985]. Our analyses indicated that the first headway (defined as the

elapsed time between green initiation and the entry of the first vehicle) and the second headway (the elapsed time between the first and second vehicle entries) each conform to Normal distributions with distinct means and variances. We found that all subsequent headways could be combined into a single Type I Gumbel distribution.

Figure 2.4. illustrates the frequency histograms for each headway "class". Table 2.2. presents the relevant statistics for each distribution along with the outcomes of the Chi-Square tests.

Table 2.2. Distribution Statistics of Discharge Headways

Headway No.	Distribution	Mean (sec)	Variance (sec^2)	χ^2_{CALC}	χ^2_{TABLE} *
First Headway	Normal	2.88	0.449	1.89	6.25
Second Headway	Normal	2.17	0.130	1.29	6.25
$\geq$ Third Headway	Type I Gumbel	1.92	0.462	9.55	10.64

*90% confidence

2.3. Initialization Times

Simulation experiments were performed to identify the amount of "simulated time" required to reach steady state operation. As the simulations begin with zero vehicles in the system, and emulated VA control responds to this initial state, relaxation times were found to be relatively large

[Hurdle, 1984]. Moreover, relaxation times were found to vary somewhat as a function of general operating conditions (e.g. demand rates).

Relaxation times for a given operating condition were identified using the techniques described by Son, Cassidy & Madanat [1994]. Namely, 12 hours of simulation were repeated 1,000 times. The numbers of queued vehicles on a given approach were measured at discrete times (i.e., at the end of each hour). Thus, the distributions of each discrete time point were identified.

Testing for steady state involved comparing the state probabilities at successive time increments for a sufficiently long time (i.e., 12 hours). Steady state conditions began at the point in time when all subsequent distributions became statistically identical. This criteria is consistent with the definition of steady state. That is, the probability distributions for the number of "customers" in the "system" at time t does not vary with time [Hall, 1991].

Thus, the occurrence of steady state conditions was tested by comparing state probability distributions over successive time positions. The smallest time position producing a distribution equivalent to all distributions generated at subsequent time positions denoted the onset of steady state. Two-way contingency table [Hogg & Tanis, 1988]

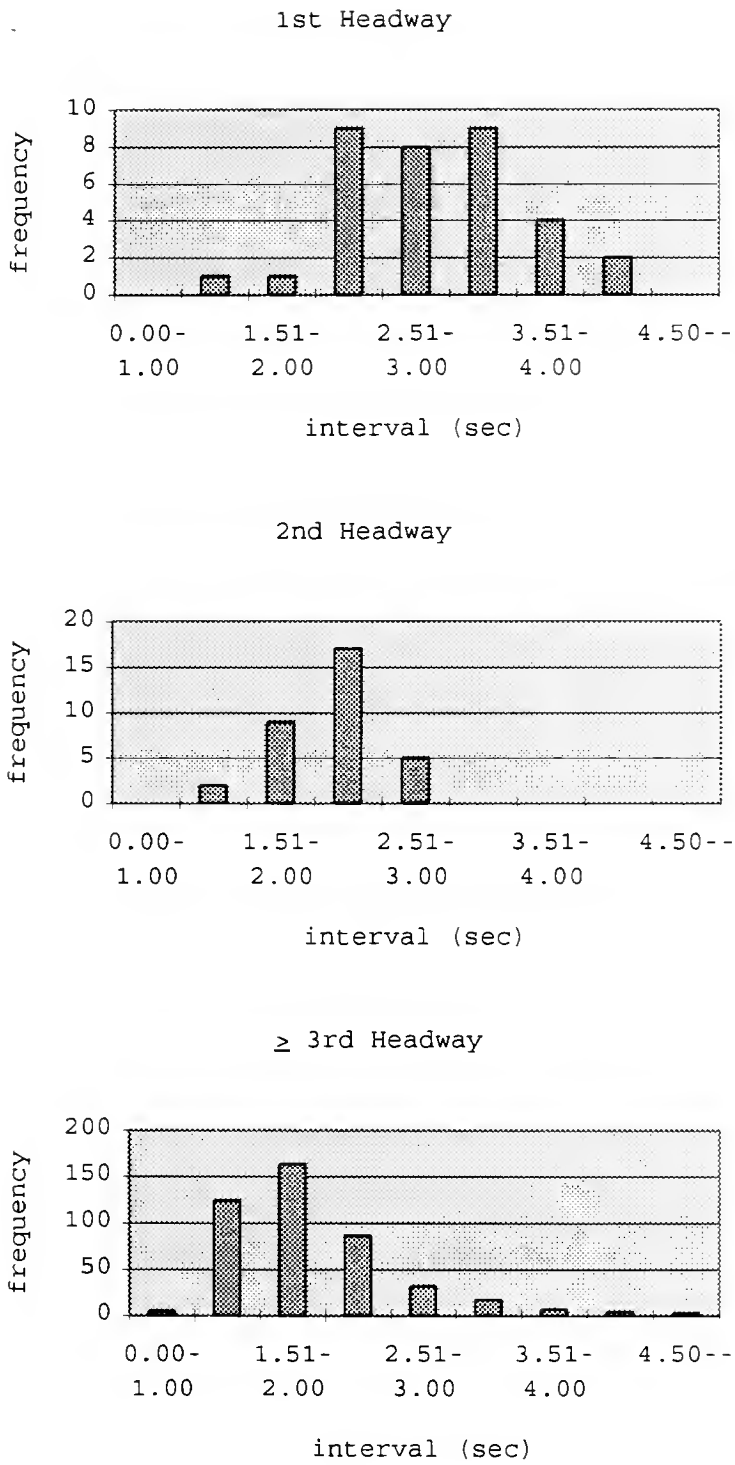

Figure 2.4. Frequency Histograms for Each Headway Class

were used for comparing the distributions across discrete time positions.

The steady state experiments described above were performed for high demand conditions so that the required initialization times identified were more than sufficient for scenarios with lower traffic flows.

2.4. Sample Size Determinations

As intersection operation (and the simulation model) is stochastic in nature, the average outcomes of repeated simulations were used to reflect performance estimates. For each scenario, the number of required simulations was identified using the expression

$$n = \left(\frac{z\sigma}{h}\right)^2$$

where n = required sample size.

z = selected confidence level (95%).

σ = sample standard deviation.

h = specified margin of sampling error for measured vehicle delay (0.4 seconds).

3. ASSESSMENT OF VA STRATEGIES

This section describes the comparative assessments of enhanced VA control strategies with more conventional practices for an array of operating conditions. Before presenting the outcomes of these assessments, we outline the desired attributes of VA control [Newell, 1988].

3.1. Desirable Attributes of VA Control

There are essentially two features of VA control which offer advantages over fixed-time signalization:

- the capability to respond to cyclic fluctuations in arrival rate.
- the capability to reduce the lost time incurred when changing signal indications.

It appears that conventional practice in deploying VA signalization fails to realize benefits obtainable from the latter control attribute. In fact, a reduction in lost time can be realized by two means:

1. efficient utilization of the change interval by discharging vehicles, and

2. reducing the required (i.e., safe and legal) duration of the change interval by only serving, to the extent possible, queued vehicles.

As Newell [1988] has pointed out, the advantage of reducing lost time stems from the resulting "chain reaction":

- the initial red phase will not be characterized by immediate queue formation;
- queues in the conflicting direction(s) are served "earlier" in the cycle; and
- the green interval returns "earlier" to the subject direction.

This "chain reaction" can promote a substantial reduction in vehicle delay.

Implementing these desirable (i.e., delay reducing) features of VA control are, to a large extent, inconsistent with conventional guidelines. For this reason, the enhanced VA strategies assessed herein may be viewed as controversial. We hold, however, that any such controversy is not founded. All enhanced strategies described and evaluated in this work are no less safe than strategies used in conventional practice. In simplest terms, any vehicle legally entitled to enter the intersection is provided with a sufficient clearance interval to do so. We elaborate on this important consideration throughout this report.

3.2. Application of Enhanced VA Strategies

Having briefly described in general terms the desirable attributes of VA control, we now demonstrate how such attributes can be achieved.

Toward deploying VA control to efficiently utilize yellow time by discharging vehicles, we note that when a vehicle queue over-runs a detector during the red phase, the signal controller identifies the number of queued vehicles requiring service only after the end of the queue passes over the detector during a subsequent green phase. The occurrence of a so-called "critical gap" (i.e., a gap or headway of sufficient length) is used to signify that the end of the queue has passed the detector.

Operating efficiencies occur when (a portion of) the yellow interval is utilized by discharging vehicles. Therefore, the optimal location for installing a detector is one in which the end of the queue is detected before the end of queue actually enters the intersection. If the clearance interval is then displayed immediately upon detecting the end of queue, a portion of the yellow interval will be characterized by queue discharge and lost time is thus reduced.

To utilize the clearance interval, the initiation of yellow would ideally occur when the end of the discharging queue is at, or perhaps just downstream of, the minimum stopping distance. The upstream edge of the detector should

therefore be separated from the intersection (e.g. the stop bar) by a distance

$$\text{Location} = \beta V_Q + S_{M(VQ)} \qquad (3.1)$$

where β = specified critical gap.

V_Q = speed of fully accelerated discharging queue (20 mph).

$S_{M(VQ)}$ = minimum stopping distance at V_Q.

Toward deploying VA control to reduce the required duration of the yellow interval, we note that an intersection approach is characterized by two "boundaries":

1. The legal stopping distance, upstream of which motorists are legally required to stop in response to the clearance interval.
2. The minimum stopping distance, downstream of which motorists will generally not stop in response to the clearance interval.

Motorists between these boundaries at the initiation of the yellow interval may elect to stop, but must be allocated sufficient yellow time should they choose to proceed through the intersection.

Knowing the vehicle velocity, V, and deceleration rate, a, a vehicle's stopping distance is expressed as $V^2/2a$. Thus, the minimum and legal stopping distances can be estimated by assuming values of deceleration which reflect 1)maximum vehicle characteristics, a_M, and 2)a "reasonable" value which drivers should be willing to tolerate, a_R. In

our work, adopted values for a_M and a_R are 10 and 8 ft/sec^2, respectively [May, 1990].

As the legal (and minimum) stopping distance varies as a function of velocity, a vehicle in a discharging queue adopts a smaller legal stopping distance than faster-moving free-flow vehicles prevailing after queue dissipation. The duration of the yellow interval should be equal to the time to travel the legal stopping distance without stopping. Thus, the required yellow time becomes $V/2a_R$ plus (perhaps) a fraction of time to accommodate driver reaction time.

This indicates that the required (i.e., safe and legal) yellow time can be reduced where all vehicles which can legally enter the intersection following yellow initiation are traveling at the reduced speeds associated with queue discharge. Hereafter, we use the variable V to denote free-flow velocity and V_Q to represent queue discharge speed (20 mph).

Where the legal stopping distance at V is downstream of the location specified by (3.1), such as at a low speed urban intersection, locating the detector as specified in (3.1) facilitates the use of a shortened clearance interval. Figure 3.1. illustrates two relevant trajectories for the "low speed" scenario. The trajectory labeled 1 represents the last vehicle in a discharging queue which crosses the detector at time t_0. Vehicle 1 is identified as the end of the queue and the yellow is displayed at time $t_0+\beta$. At time

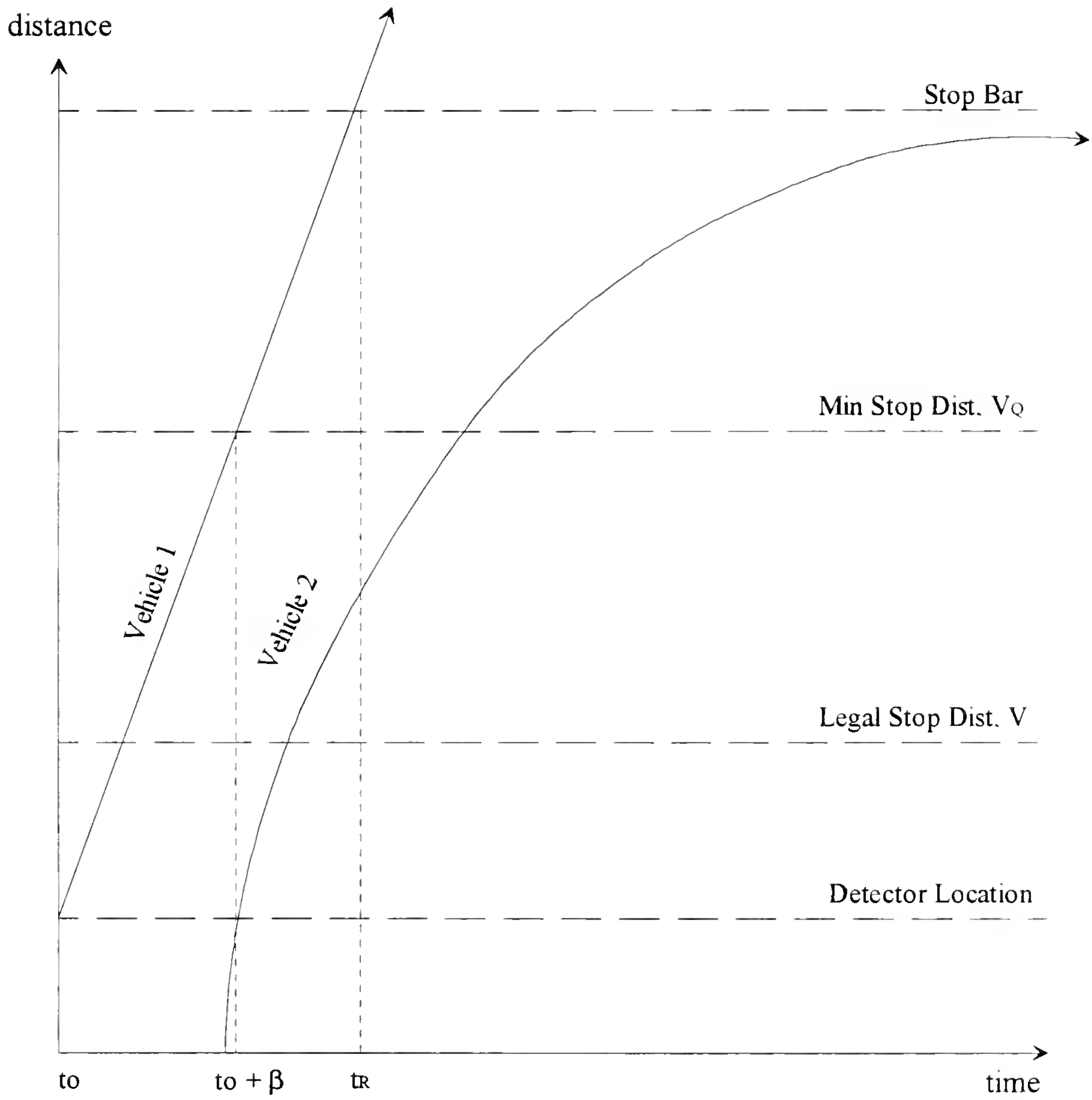

Figure 3.1. Trajectories Depicting Enhanced VA Strategy, Low Speed Intersection (Source: Newell, 1988)

$t_0+\beta$, vehicle 1 is at its minimum stopping distance and will therefore (generally) proceed through the intersection.

The end of the clearance interval can occur at t_R when, or perhaps moments after, vehicle 1 is projected to enter the intersection. Any vehicle arriving after $t_0+\beta$ is upstream of the legal stopping distance, $V^2/2a_R$, at the yellow initiation and is therefore legally required to stop. Extending the yellow to a duration of $V/2a_R$ (as one would have to do with a fixed-time signal) represents nothing more than "wasted time."

The above strategy guarantees that 1) the yellow interval is, on average, effectively utilized by discharging vehicles and 2) the clearance time is not extended longer than necessary.

During any cycle when the queue does not over-run the detector during the red phase, we adopt a signal timing strategy similar to, but slightly more efficient than, policies used in conventional practice. An initial green is allocated to accommodate queued vehicles counted by the detector(s) during the red phase. The green interval is extended (by 2 seconds) with each subsequent arrival during the green. The clearance interval is displayed when no additional extensions are "called" or when the maximum green time has elapsed.

For our work, the duration of the yellow interval displayed under these circumstances is always sufficient to

safely accommodate all vehicles legally entitled to enter the intersection. The details of these yellow time allocations are later described in their example applications.

Enhanced VA strategies are now presented by means of example applications. We initially demonstrate and quantify the benefits of enhanced strategies using intersections formed by two one-way streets. Although such scenarios are highly idealized (and infrequently occur in practice), these simplified conditions serve to illustrate the potential impacts of enhanced control policies. Once these benefits are illustrated, the VA strategies tested on two-way intersections are exploited for four-way intersections.

3.2.1. Intersection of Two One-Way Streets, One Lane Per Approach

To illustrate the potential benefits of the enhanced VA strategies just described, we first adopt the following idealized scenario:

- The intersection geometrics consist of two one-way streets with one (through) lane on each approach, as illustrated in Figure 3.2..
- The specified critical gap, β, is 3.5 seconds.
- The maximum green time is 60 seconds for each direction.

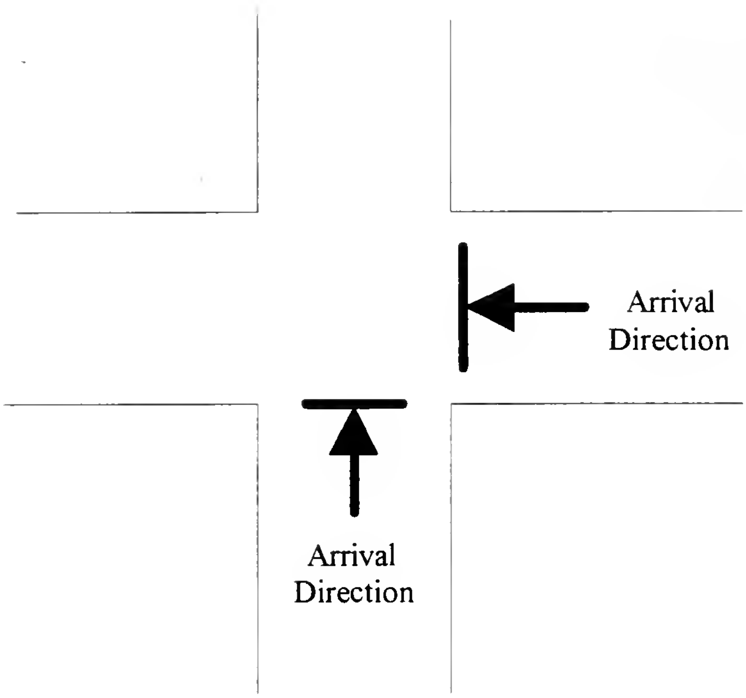

Figure 3.2. Intersection of One-Way Streets, One Lane Per Approach

<u>**3.2.1.1. Impulse Detectors -- Low Speed Intersection**</u>

We first assess performance using impulse detectors at a signalized intersection exhibiting a low average free-flow vehicle speed of 30 mph. An impulse detector, located upstream of the stop bar, measures the elapsed time between successive vehicle arrivals. An interarrival time greater than β (3.5 seconds in this scenario) triggers the yellow initiation.

For all scenarios involving the use of impulse detectors, a sufficient minimum green time is provided so that the "start-up" wave of queued vehicles passes over the

detectors. This feature helps to safeguard against premature green termination.

Conventional Strategy:

The impulse detectors are located 96 ft upstream of the stop bar, consistent with the distance suggested in conventional traffic signal handbooks [Kell & Fullerton, 1991]. This suggested distance facilitates intersection entry to the entire queue prior to initiating the clearance interval.

The yellow duration is 3 seconds to accommodate free-flow vehicles which might be just downstream of the legal stopping distance at the initiation of yellow.

Enhanced Strategy:

The impulse detectors are located 146 ft upstream of the stop bar, consistent with the distance specified in (3.1).

As the legal stopping distance for free-flow vehicles (121 ft) lies downstream of the detectors, the yellow time can be reduced to 2 seconds to accommodate vehicles discharging from queue at speed V_Q (20 mph). The duration of the yellow time is extended to 3 seconds for safety reasons only during cycles where the queue does not over-run the detector and subsequent

vehicle arrivals extend the green interval to its maximum value (60 seconds).

Table 3.1. presents the simulated outcomes for the scenarios described above. Performance is assessed under directional demand rates of 800 vph and 480 vph. To illustrate the impacts of enhanced VA strategies, we have independently simulated the operation resulting from 1) moving the detectors to the upstream location specified in (3.1) without shortening the clearance interval and 2) locating the detectors as in (3.1) and reducing the yellow time to 2 seconds.

Table 3.1. Simulated Outcomes, Two-Way Intersection, Low Speed, Impulse Detector

	Conventional Strategy		Enhanced Strategy Yellow=3secs		Enhanced Strategy Yellow=2secs	
Directional Demand (vph)	800	480	800	480	800	480
Average Delay (secs)	21.4	6.8	18.3	5.8	16.1	5.7
% Reduction	--	--	14.5	14.7	24.8	16.2

As noted in Table 3.1., merely moving the detectors upstream (without reducing the clearance interval) reduces average vehicle delay. Shorting the yellow time results in further delay savings.

Delay reductions are less dramatic under low demand conditions (i.e., 480 vph per direction). This is a consequence of the infrequent occurrence of queues over-running the detector under low flows.

3.2.1.2. Loop Detector -- Low Speed Intersection

Loop detectors measure the presence of vehicles and terminate green time when the loop is no longer occupied. We now assess the impacts of deploying loop detectors to the operating scenario just described.

Conventional Strategy:

A loop detector 103 ft in length is installed so that its downstream edge is at the stop bar, consistent with conventional practice. A 3-second clearance interval occurs when the loop is no longer occupied.

Enhanced Strategy:

Under the enhanced scheme, the loop detector is located using a strategy similar to that used for the impulse detector. The length of the loop is βV_Q (103 ft) so that no occupancy on the loop is equivalent to observing a critical gap of β. The downstream edge of the loop is placed at the minimum stopping distance for speed V_Q. A short minimum green time of 4 seconds is used to accommodate the rare occurrence of a queue not

reaching the loop detector. A yellow time of 2 seconds (to accommodate vehicles discharging in queue) is initiated when the loop becomes unoccupied. A 3-second yellow time is used when the maximum green time occurs.

Table 3.2. presents delays under both the conventional and the enhanced strategies. For high demand conditions, the conventional use of loop detectors results in greater delay than the conventional deployment of impulse detectors. This is because locating the downstream edge of the loop at the stop bar virtually guarantees that no portion of the clearance interval will be used by discharging vehicles (except perhaps where the maximum green time elapses).

Table 3.2. Simulated Outcomes, Two-Way Intersection, Low Speed, Loop Detector

	Conventional Strategy		Enhanced Strategy	
Directional Demand (vph)	800	480	800	480
Average Delay (secs)	33.4	6.3	13.9	3.8
% Reduction	--	--	58.4	39.7

The enhanced strategy for loop deployment yields delay savings which exceed those generated by impulse detectors. This is because the location of the loop minimizes the

occurrence of queues which do not over-run the detector. As such, the need for green time extensions is rare.

3.2.1.3. Impulse Detector -- High Speed Intersection

With respect to the detector placement strategy, a complication occurs when high free-flow speeds prevail. As speed V increases, the legal stopping distances for free-flow vehicles lie upstream of the location described by (3.1). If the detector is placed as specified in (3.1), an arrival headway exceeding β does not guarantee that a vehicle traveling at free-flow speed V will be upstream of its legal stopping distance at the initiation of yellow. As such, an extended yellow time of duration $V/2a_R$ would be required for safety reasons.

Moving the detectors upstream to the legal stopping distance at speed V, $V^2/2a_R$, would mean that the end of a discharging queue would be upstream of its minimum stopping distance at the initiation of yellow. In fact, for high approach speeds such as 50 mph, the end of queue might even be upstream of its legal stopping distance at the onset of yellow. As such, the clearance interval might create some degree of residual queuing.

Locating the detector at the legal stopping distance for V would, however, facilitate a shorter yellow time by serving only vehicles traveling at V_Q. An arrival headway exceeding β guarantees that all free-flow vehicles

(traveling at V) are upstream of the legal stopping distance at yellow initiation. A preferable VA strategy might therefore be to insure that the last vehicle to enter the intersection travels at speed V_Q, rather than to guarantee that the entire queue is served during the green phase. We demonstrate this proposition with the following scenarios:

- The intersection's simple geometrics are as illustrated in Figure 3.1.
- High approach speeds prevail (40 mph and 50 mph).
- The critical gap, β, is 4.0 seconds.
- The maximum green time is 60 seconds for each direction.
- Impulse detectors are deployed.

Conventional Strategy:

The detector is located 160 ft upstream of the stop bar as specified by (3.1).

The yellow duration is $V/2a_R$ to safely accommodate free-flow vehicles.

Enhanced Strategy:

The detectors are located at the legal stopping distance for free-flow vehicles, $V^2/2a_R$.

During cycles when the queue over-runs the detector by the initiation of green, the yellow duration is 2 seconds to accommodate vehicles traveling at V_Q.

When the queue does not over-run the detector, the displayed yellow time plus the 2-second green extension is equal to the required clearance time for free-flow vehicles, $V/2a_R$. If green time extensions continue until the maximum green (60 seconds) elapses, the yellow time displayed has a duration of $V/2a_R$. As such, all vehicles legally entitled to enter the intersection do so without encountering the red phase.

Table 3.3. presents the simulated outcomes for the "high speed" scenarios described above. The enhanced VA strategies do facilitate fairly dramatic delay reductions. As expected, the proposed enhancements are more effective for the lower free-flow speed of 40 mph as the detector placement results in a reduced tendency to "cut-off" discharging queues.

Table 3.3. Simulated Outcomes, Two-Way Intersection, High Speed, Impulse Detector

	V = 40 mph				V = 50 mph			
	Conventional Strategy		Enhanced Strategy		Conventional Strategy		Enhanced Strategy	
Directional Demand (vph)	800	480	800	480	800	480	800	480
Average Delay (secs)	23.7	7.3	18.0	4.0	25.8	8.1	22.4	5.4
% Reduction	--	--	24.0	45.2	--	--	13.2	33.3

3.2.2. Multilane Intersections

In practice, a VA signal typically controls more than one traffic lane per approach. From our observations, it appears that most, perhaps all, VA signals on multilane approaches treat the multilane traffic streams as if they are a single traffic stream. Detectors, which might be separately installed in individual lanes, are "wired together" such that multiple detectors function as a single detector covering all lanes.

Following queue dissipation, vehicles approach the intersection at the given arrival rate -- a rate which is generally much lower than the queue discharge rate. However, arriving vehicles traveling "side-by-side" in adjacent lanes may exhibit "side-by-side" headways equivalent to those of a discharging queue in a single lane. As the detectors are superimposed across all travel lanes, the relatively small side-by-side headways are interpreted by the controller as discharge headways occurring in an individual lane. Simply stated, the detectors are unable to identify queue dissipation. Green time is therefore terminated (prior to the maximum green) only if a critical gap β occurs simultaneously in all travel lanes.

An enhancement to this strategy is to search for gaps in individual lanes. Once a critical gap β occurs in any given lane, the green interval could be terminated. Or, if one is concerned that imbalanced lane use might cause queues

to be "cut-off", green time could be terminated after a critical gap β occurs once in each lane. That is, once β occurs in a given lane, the controller should no longer "poll" that lane for the remainder of the green interval. With this latter strategy, additional yellow time may be required to accommodate free-flow vehicles traveling in lanes where queues previously dissipated.

We illustrate potential impacts of this enhanced VA strategy using the following scenario:

- The intersection is formed by two one-way streets with two (through) lanes on each approach as illustrated in Figure 3.3.

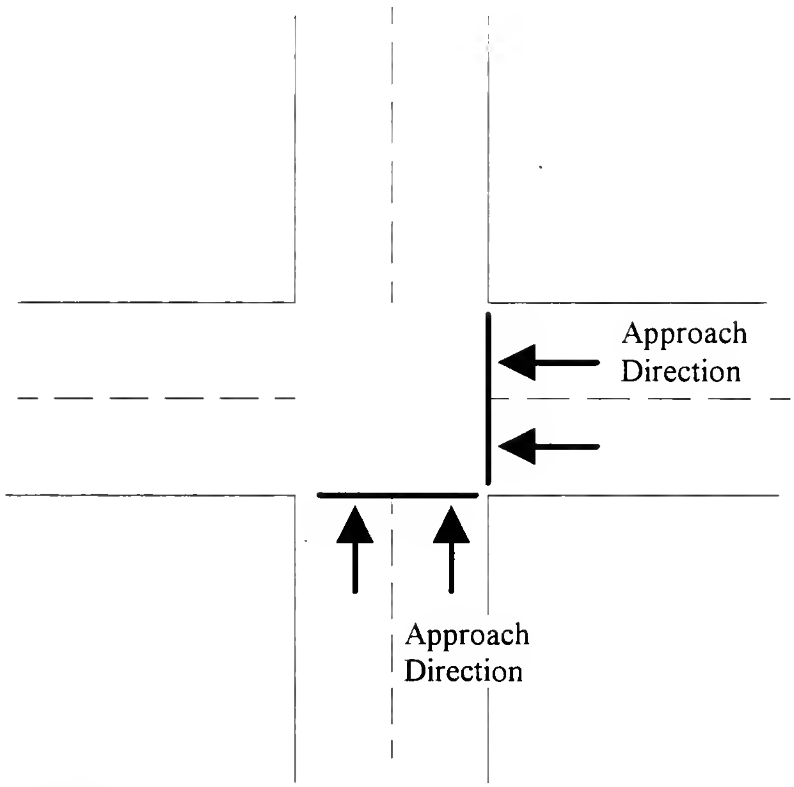

Figure 3.3. Intersection of One-Way Streets, Multilane Approaches

- Impulse detectors are deployed with a critical gap, β, of 4.0 seconds.
- The maximum green time is 60 seconds for each direction.
- Performance is simulated under free-flow speeds of 35 mph and 50 mph. With the lower speed of 35 mph, the detectors are located as (3.1), upstream of the legal stopping distance at V. For approach speeds of 50 mph, the detectors are located at the legal stopping distance, $V^2/2a_R$.
- Given the above strategy for locating detectors, the yellow duration is 2 seconds (whenever queues over-run the detectors).
- Performance is simulated under directional demand rates of 1,600 vph and 960 vph.

Conventional Strategy:

The detectors are "wired together" so that the green interval is terminated (prior to maximum green) only when β is observed simultaneously across both approach lanes.

Enhanced Strategy:

Detectors search for β in individual lanes. The green time is terminated (prior to maximum green) when β occurs in either lane.

Table 3.4. presents simulated outcomes for the multilane scenarios above. We note that the "conventional" strategies assessed are optimal in every sense except that gaps are simultaneously sought across all lanes. For lower approach speeds, the enhanced strategy for multiple lanes substantially reduces average delay. For higher free-flow speeds, the enhanced strategy actually erodes performance. This is a consequence of cutting-off queues created by the combined influence of moving the detectors upstream *and* terminating green when only one queue has passed over the detector. This problem can likely be rectified by deploying the slightly more sophisticated strategy of initiating yellow when a critical gap occurs once in each lane.

Table 3.4. Simulated Outcomes, Two-Way Intersection, Multilane Approaches, Impulse detector

	V = 35 mph				V = 50 mph			
	Conventional Strategy		Enhanced Strategy		Conventional Strategy		Enhanced Strategy	
Directional Demand (vph)	1600	960	1600	960	1600	960	1600	960
Average Delay (secs)	33.5	7.0	26.5	6.6	35.3	8.5	37.6	8.5
% Reduction	--	--	20.4	5.7	--	--	-6.5	0.0

3.2.3. Four-Way Intersections

Although highly idealized, the two-directional intersection scenarios assessed thus far serve to illustrate

potential benefits obtained by deploying VA control to facilitate

1. efficient utilization of the clearance interval,
2. a reduction in yellow time by serving only queued vehicles, and
3. the detection of gaps in individual traffic lanes.

Having demonstrated the delay reduction achievable by exploiting the above attributes, we now incorporate these features in VA control for four-way intersections.

In general, VA signals are used to simultaneously control opposing travel directions as in Figure 3.4. This situation is more complicated than the aforementioned two-directional scenarios in that switching the signal from green to yellow, and from yellow to red, is influenced by queue evolution in both opposing directions.

Some conventional strategies terminate green (prior to the maximum green) only when β occurs simultaneously across all lanes and in all intersection approaches controlled by the green interval. Apparently as an attempt to promote "snappier" operation, so-called volume-density control is commonly employed. With this conventional strategy, the specified critical gap, β, gradually decreases over time.

These conventional schemes do not exploit the desired control attributes which we have demonstrated to reduce delay. The strategies

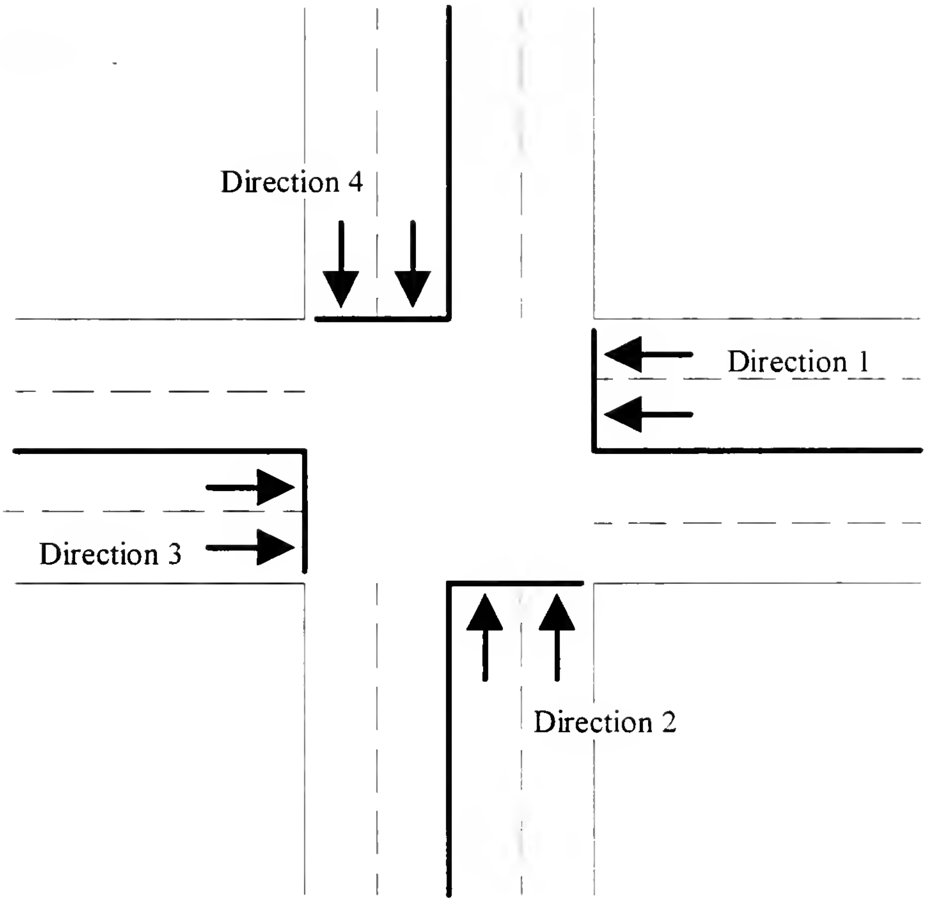

Figure 3.4. Four-Way Intersection, Multilane Approaches

1. do not efficiently utilize the yellow interval;
2. treat multiple traffic streams as if they were a single traffic stream; and
3. do not exploit a reduced clearance interval.

Exploiting the proposed attributes at a four-way intersection can become complicated as the presence of opposing traffic may dictate the required (i.e., safe) yellow duration. That is, if the yellow initiates in

response to queue dissipation in the "high demand" direction, an opposing vehicle traveling at free-flow speed V may be just within the legal stopping distance, thus requiring an extended yellow duration of $V^2/2a_R$.

With this consideration in mind, four-way intersections are grouped into two categories:

- Unbalanced directional demand rates, i.e., queues generally dissipate in "low demand" directions before dissipating in "high demand" directions.
- Balanced directional demand rates, i.e., green times needed are nearly equal for both opposing directions.

3.2.3.1. Unbalanced Directional Demands

For the case of unbalanced directional demands, the following enhanced VA strategy is evaluated:

> Detectors are located (in individual lanes) at a distance specified by (3.1) or by the legal stopping distance at free-flow speed, V, whichever is larger. Once the queues in the low demand direction dissipate, the detectors on that approach continue to record subsequent arrival times of vehicles past the detectors. The yellow interval is displayed when a critical gap, β, is observed in any lane on the high demand approach. The controller immediately identifies the most recent arrival time on the low demand approach and allocates sufficient yellow time to serve this

vehicle or provides enough yellow to serve the discharging queue in the high flow direction, whichever is larger.

To illustrate the potential benefits of this proposed VA strategy, we examine the following intersection conditions:

- A four-way intersection with two (through) lanes on each approach as illustrated in Figure 3.4.
- Demand on each high flow approach (directions 1 and 2 in Figure 3.4.) is 1,600 vph. Demand in opposing directions 3 and 4 is 960 vph.
- Free-flow average speed, V, is 40 mph.
- Maximum green time is 60 seconds for each direction.
- Impulse detectors are deployed.

Conventional Strategy, fixed β:

The detectors are located 160 ft upstream of the stop bar, consistent with (3.1). As this location lies downstream of the legal stopping distance at speed V, the yellow duration is 4 seconds.

Green time is terminated (prior to maximum green) when a critical gap, β, of 4.0 seconds is observed simultaneously across all lanes in both opposing directions.

Conventional Volume-Density Control:

The detectors are located 160 ft upstream of the stop bar and the yellow duration is 4 seconds.

Green time is terminated (prior to maximum green) when a critical gap, β, is observed simultaneously across all lanes in both directions. The specified value of β changes over time. During the initial 30 seconds of green, β is 4.0 seconds. The value of β sequentially decreases by 0.15 seconds each 1.5 seconds of green until reaching a minimum value of 2.5 seconds.

Enhanced Strategy:

The detectors are located at the legal stopping distance for V (215 ft upstream of the stop bar).

Green time is terminated (prior to maximum green) when a critical gap of 4.0 seconds is observed in either lane on the high demand approach.

The yellow interval displayed each cycle is of sufficient duration to accommodate vehicles traveling at speed V_Q (20 mph) on approach 1 or 2 or to provide entry to free-flow vehicles which most recently passed detectors on approach 3 or 4, whichever is larger.

When queues in directions 1 or 2 do not over-run the detector during the red phase, yellow time is allocated to provide legal and safe intersection entry based upon measured arrival times past the detectors.

Table 3.5. presents the outcomes for each of the three VA strategies described above. The enhanced strategy results in significantly less delay than those generated from conventional strategies.

Table 3.5. Simulated Outcomes, Four-Way Intersection, Unbalanced Demands, Impulse Detector

	Conventional Strategy fixed β = 4.0 secs	Conventional Volume-Density Control	Enhanced Strategy
Average Delay (secs)	34.0	34.4	24.9
% Reduction	--	-1.2	26.8

3.2.3.2. Balanced Directional Demands

Where demands in opposing directions approach the same magnitude, natural fluctuations in arrival rates will vary the direction in which queues dissipate first. As such, the direction requiring longer green time will change from cycle to cycle. The following VA strategy is evaluated for balanced conditions:

> Green time is continued until queues in both directions dissipate, provided the resulting green interval does not exceed some *initial* specified maximum value. This initial maximum green duration might be, for example, slightly larger than the optimal value for a fixed-time signal.

Once the initial maximum value is reached, the green interval terminates only if the queue has previously dissipated on the approach exhibiting slightly higher demand. If the initial maximum green has elapsed and queues persist in the high demand direction, green is extended until queues in the high demand direction dissipate or until the green interval reaches the specified maximum acceptable value (60 seconds).

The above strategy may occasionally create residual queuing. However, these queues will be served in subsequent cycles.

As described earlier in section 3.2.3., the yellow duration is established each cycle to accommodate vehicles traveling at speed V_Q or to provide entry to free-flow vehicles which most recently passed detectors (on either approach), whichever is larger.

To demonstrate the potential benefits of the enhanced VA strategy, we examine the following scenario:

- A four-way intersection with two through lanes on each approach as illustrated in Figure 3.4.
- Demand in direction 1 and direction 2 is 1,600 vph. Each of the opposing directions exhibit a slightly lower demand rate of 1,500 vph.
- Free-flow speed, V, is 40 mph.
- Maximum green time is 60 seconds for each direction.
- Impulse detectors are deployed.

We evaluate the above scenario under three control strategies. The first two are 1) conventional VA control with fixed β and 2) conventional volume-density control. These conventional strategies are identical to those previously evaluated for unbalanced directional demands.

For the enhanced VA strategy, we adopt the control scheme previously described for balanced directional demands. The "initial" maximum green time (for both directions) is 45 seconds.

Table 3.6. presents the outcomes from the three control strategies described above. The enhanced VA strategy reduces delay over conventional schemes.

Table 3.6. Simulated Outcomes, Four-Way Intersection, Balanced Demands, Impulse Detector

	Conventional Strategy fixed $\beta = 4.0$ secs	Conventional Volume-Density Control	Enhanced Strategy
Average Delay (secs)	38.3	34.5	26.3
% Reduction	--	9.9	31.3

4. CONCLUSIONS

This report has illustrated the potential benefits (i.e., reduced vehicle delay) obtained through enhanced VA control strategies. These enhanced strategies seek to 1) facilitate the use of the clearance interval by discharging vehicles, 2) shorten the duration of the required (i.e., safe) clearance interval by only serving, to the extent possible, queued vehicles and 3) evaluate gaps in individual traffic streams. Although dramatically inconsistent with current guidelines/practice, the enhanced strategies described herein insure that a sufficient yellow duration is provided to all motorists legally entitled to enter the intersection. The enhanced VA strategies will therefore not significantly increase the incidence (i.e. the frequency) of motorists "illegally" entering the intersection. Reducing the clearance interval can, however, exacerbate the severity of any "illegal" intersection entry. That is, illegal intersection entries may occur at a later time relative to termination of the yellow interval.

This potential concern can be remedied by utilizing additional loop detectors located downstream of the "primary" detectors (i.e., at or near the intersection stop

bar). The "primary" or upstream detectors would be used to identify optimal times for switching signal indications from green to yellow, as described throughout this report. The downstream loop detectors can improve safety by extending the yellow interval whenever

1. A motorist legally entitled to enter the intersection is forced to decelerate (e.g. due to slow-moving vehicles downstream) yet opts to enter the intersection, and
2. A motorist not legally entitled to enter the intersection elects nonetheless to do so.

Downstream loop detectors would further promote safety by providing the system with a certain level of redundancy. Such redundancy would be valuable when upstream detectors fail to detect a vehicle (or when the detectors are malfunctioning). Moreover, exploiting downstream loop detectors to dictate termination of the clearance interval could further improve performance by initiating the red indication immediately after the final vehicle, which can and will enter the intersection, actually does so.

The benefits of exploiting downstream detectors were not explored in this study. The scope of this work has not been to investigate every possible control strategy (under countless numbers of prevailing conditions). Rather, the objective has been to assess a "hand-full" of representative examples to illustrate possible advantages associated with improved VA control schemes.

Surprisingly, the enhanced VA strategies do not appear to increase the extent to which vehicles are required to stop at the intersection. Table 4.1. presents the average percentage of vehicles required to stop for the four-way intersection scenarios described in section 3.2.3. The enhanced strategies slightly decrease the percentage of stops at the intersection.

The enhanced strategies seek to serve only queued vehicles, which would normally increase the percentage of stopped vehicles in the traffic stream. The marginal reductions displayed in Table 4.1. are likely the consequence of moving the detectors upstream to the legal stopping distance for vehicles traveling at free-flow speed *V*. By placing the detectors relatively far upstream of the stop bar, queues over-ran the detectors by the initiation of green in less than 40 percent of the cycles. The green time allocation strategies when queues do not over-run the detector do not promote exclusive service to queued vehicles.

Table 4.1. Simulated Outcomes, Four-Way Intersection, Percent Stopping

	Unbalanced Directional Demands			Balanced Directional Demands		
	Conventional Strategy $\beta = 4.0$ secs	Conventional Vol-Density Control	Enhanced Strategy	Conventional Strategy $\beta = 4.0$ secs	Conventional Vol-Density Control	Enhanced Strategy
% Stopping	0.83	0.84	0.81	0.91	0.90	0.89
% Reduction	--	-1.2	2.4	--	1.1	2.2

The authors hold that the dramatic delay reductions resulting from these enhanced strategies should, perhaps at the very least, motivate the traffic engineering profession to re-examine its policies and guidelines concerning the application of VA signal control.

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